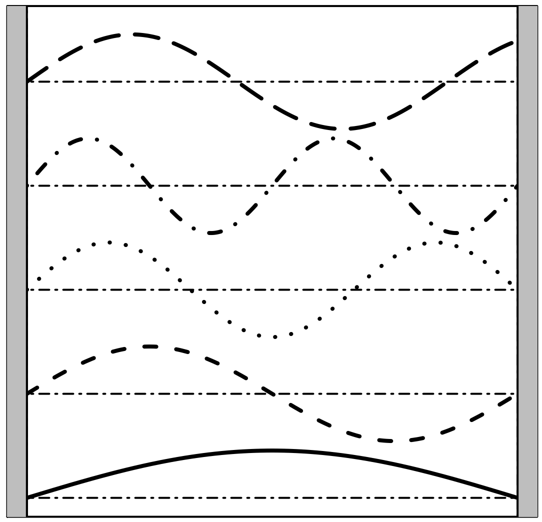
Key for The Wave Model of Electrons

The figure below shows five examples of sine functions with the general format of $y = \sin(n \times x)$ over the range $x = 0$ to $x = \pi$. The values of n, from the bottom to the top, are $n = 1$, $n = 2$, $n = 3$, $n = 4$, and $n = 2\frac{1}{3}$. For each sine wave, the horizontal line shows where $y = 0$. Note that the sine waves are offset from each other along the y-axis so they are easier to examine individually.



Examine this set of sine waves. In what way(s) are they similar to each other?

> All five sine waves begin with an amplitude of zero. All five sine waves have a positive amplitude for some portion of the wave. All five sine waves have the same value for their maximum amplitude.

In what way(s) are they different from each other?

> Each wave has a different number of peak maxima or minima (1 for the $n = 1$, 2 for $n = 2$, 3 for $n = 3$, etc.) Each wave crosses the line where $y = 0$ a different number of times (0 for $n = 1$, 1 for $n = 2$, etc). Four of the sine waves have a final value of zero; the wave for $n = 2\frac{1}{3}$ does not.

When we see a pattern it often indicates something interesting is going on. What pattern is there between n and the number of times the sine wave changes its sign? We call a point where the sign changes a node. What is the value of y at a node?

> The number of nodes is equal to $n - 1$, although for the case where $n = 2\frac{1}{3}$ we have to round up to 3. The amplitude (the value of y) is 0 at a node.

The gray bars on the sides of the figure mark boundaries where the sine wave ceases to exist; that is, the sine wave is confined to the area between the boundaries. Restricting a sine wave in this way is not unusual. A vibrating guitar string, for example, has boundaries where it is anchored to the bridge and where your finger anchors it to the fretboard. Using the figure above and the equation $y = \sin(n \times x)$, what must be true to ensure that $y = 0$ at the two boundaries?

> Any value of n that is a positive integer greater than zero; thus the allowed values are $n = 1, 2, \ldots$

The equation $y = \sin(n \times x)$ describes a wave confined to move in one dimension, x, and includes a single variable, n, that is restricted to positive integers ($n = 1, 2, 3, ...$) when boundary conditions exist. We call this a quantized system. Let's consider an electron as a wave. As is the case with our simple sine wave model, an electron has boundaries because it is confined to the atom; thus, it is quantized. Because the electron can move in three directions, x, y, and z, its equation will include three variables—which we call quantum numbers—each with restrictions on its allowed values. These quantum numbers are n, l, and m_l. The table below gives the allowed values for $n = 1$, $n = 2$, and $n = 3$:

n	l	m_l	n	l	m_l	n	l	m_l
1	0	0	2	0	0	3	0	0
			2	1	$0, \pm 1$	3	1	$0, \pm 1$
						3	2	$0, \pm 1, \pm 2$

What pattern(s) can you find in the values for these quantum numbers? Can you write a simple set of rules that explain this pattern?

> The allowed values for n are $n = 0, 1 \ldots$ The allowed values for l are $l = 0, 1 \ldots n - 1$. The allowed values for m_l are $m_l = 0, \pm 1 \cdots \pm m_l$. For any value of n there are n possible values of l, and for any value of l there are $2 \times l + 1$ possible values for m_l.

Given your set of rules, list the complete set of quantum numbers for $n = 4$.

> For $n = 4$, the following sets of quantum numbers are possible

n	l	m_l
4	0	0
4	1	$0, \pm 1$
4	2	$0, \pm 1, \pm 2$
4	3	$0, \pm 1, \pm 2, \pm 3$

Given your set of rules, what value(s) of n are possible if $m_l = +4$?

> An m_l of 4 requires an l of 4, which, in turn, requires that $n \geq 5$.