

*measurement exercise x: probability, patterns, and randomness*

**Exercise 1.** Guess the rule I use to create sequences of three numbers. I will provide you with a sequence that obeys my rule; your goal is to guess my rule. To gather information, write down a sequence of three numbers and I'll tell you if it obeys or if it does not obey my rule. When you think you know my rule, write it down and I'll tell you if it is correct or if it is incorrect. Each test sequence or correct statement of my rule counts as one point; each incorrect statement of my rule counts as five points. Your goal is to find my rule while accumulating the smallest number of points.

**Exercise 2.** Suppose you roll a fair six-sided die 12 times, each time recording the number shown on the top of the die. Which of the following sequences is the most likely outcome? Which is the least likely outcome? Why?

2 2 1 3 1 5 4 4 6 1 4 1

1 1 1 1 1 1 1 1 1 1 1 1

5 4 1 6 4 6 1 4 2 5 6 4

**Exercise 3.** On the game show *Lets Make a Deal*, the host, Monty Hall, ended the show by offering a member of the audience an opportunity to win a car. The audience member, let's make it you, is presented with three doors: behind one door is a car; behind the other two doors are goats. You select a door. Monty then opens one of the two remaining doors, always choosing one that contains a goat. Monty asks if you want to keep your original choice of door or switch to the other unopened door. Should you switch doors or remain with your original choice? Explain your decision?

**Exercise 4.** Suppose an urn is filled with 100 red balls and 100 green balls. While blindfolded, you select two balls from the urn at random, placing the first ball in your closed left hand and the second ball in your closed right hand. Without looking at the ball in your left hand, what is the probability that the second ball selected, the one in your right hand, is red? How did you arrive at your answer?

**Exercise 5.** Shown below are the first 100 results—ordered by row, beginning in the upper left corner and ending at the lower right corner—from a machine that spits out a number from 1 to 6 every minute. The house offers you the following bet: guess the next 10 results produced by the machine, with a payout of \$10 for each correct guess against a loss of \$9 for each incorrect guess. What are your guesses? How did you make a decision about your guesses?

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]
[1,]	3	3	3	1	5	3	1	5	6	1
[2,]	2	4	3	3	6	3	1	1	1	3
[3,]	4	3	3	3	5	3	1	3	3	3
[4,]	3	4	3	3	3	3	1	3	3	4
[5,]	2	3	2	4	3	3	3	5	2	3
[6,]	3	6	2	3	4	1	2	3	3	1
[7,]	3	4	3	5	6	3	1	3	3	6
[8,]	2	1	4	3	3	5	3	2	3	5
[9,]	3	4	3	3	3	6	1	3	3	5
[10,]	1	3	1	6	3	6	6	6	5	3

**Exercise 6.** Suppose we flip a fair coin 20,000 times, keeping track after each flip of the cumulative number of heads and tails. For example, the first 10 flips might be H H T H T H T T H H. In this case, the cumulative number of heads and of tails after two throws is two heads and zero tails; after ten flips it is six heads and four tails. During the course of these 20,000 flips, how often will the cumulative number of heads and of tails be equal to each other? Approximately 25 times? 50 times? 250 times? 500 times? 2500 times? 5000 times? 10,000 times? How did you make a decision?

**Exercise 7.** Suppose we repeat the game in Exercise 6 a thousand times, keeping track, each time, of how many often the cumulative number of heads and of tails equal each other. How frequently will we find that the cumulative number of heads and of tails never equal each other? 1000 out of every 1000 games? 100 out of every 1000 games? 10 out of every 1000 games? 1 out of every 1000 games? Even less frequently? How did you make a decision?

**Exercise 8.** You are about to register for classes. Which of these is the more likely outcome of your initial pass through registration? Why?

- you are not placed in any courses, or
- you are not placed in any courses because you picked only courses that are not open to first-year students (coded as priority 0 = FR).

**grid for exercise 1**

cycle	test sequence or statement of my rule	correct or incorrect	points
1	1, 2, 4	correct	1
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			