Key for Review of Basic Mathematics

An important skill in Chem 260 is the ability to rearrange mathematical expressions to isolate a variable of interest; these two questions provide practice in this:

1. Rearrange the following equation by solving for a in terms of the other variables; your final equation should be in the form $a = ...$ with any fractions written in their simplest form.

$$\left(a + \frac{b}{c}\right) \times (d - e) = f$$

First, let's divide both sides of the equation by $(d - e)$

$$\left(a + \frac{b}{c}\right) = \frac{f}{d - e}$$

and then subtract $\frac{b}{c}$ from both sides of the equation to arrive at the final answer

$$a = \frac{f}{d - e} - \frac{b}{c}$$

2. Rearrange the following equation by solving for c in terms of the other variables; your final equation should be in the form $c = ...$ with any fractions written in their simplest form.

$$a = b\left(\frac{1}{c} - \frac{1}{d}\right)$$

First, let's divide both sides of the equation by b

$$\frac{a}{b} = \left(\frac{1}{c} - \frac{1}{d}\right)$$

and then add $\frac{1}{d}$ to both sides of the equation

$$\frac{a}{b} + \frac{1}{d} = \frac{1}{c}$$

This leaves us with $\frac{1}{c}$ on the left side of the equal sign; we want to write this in terms of c, so we take the reciprocal of both sides, arriving at

$$c = \frac{1}{\frac{a}{b} + \frac{1}{d}}$$

which we clean up by multiplying the right side of the equation by bd/bd

$$c = \frac{bd}{ad + b}$$

To gain comfort with natural and base 10 logarithms, determine the value of x to two decimal places for each of the following four problems.

3. $\log(x) = 0.83$

To determine the value of x we take the inverse log of both sides of the equation. Depending on your calculator, you may accomplish this by entering 0.83 and selecting INV LOG or by selecting 10^x; in either case, the value of x is 6.76.

4. $x = \log(0.0135)$

   To determine the value of x, enter 0.0135 into your calculator and select the LOG key (not the LN key, which is for base e). The value of x is -1.87.

5. $\ln(x) = 0.122$

   To solve for x, we take the inverse natural log of both sides of the equation. Depending on your calculator, you may accomplish this by entering 0.122 and selecting INV LN or by selecting e^x; in either case, the value of x is 1.130.

6. $x = \ln(1.83)$

   To determine the value of x, enter 1.83 into your calculator and select the LN key (not the LOG key, which is for base 10). The value of x is 0.604.

We will work with quadratic equations later this semester when solving equilibrium problems; these two questions provide practice in working with quadratic equations:

7. Rearrange the following equation into the form $ax^2 + bx + c = 0$.

   $$0.20 = \frac{x^2}{55 - x}$$

   We begin by mulitplying both sides of the equation by $55 - x$

   $$x^2 = 0.20\,(55 - x)$$

   and then multiply the 0.20 through the right side of the equation

   $$x^2 = 11 - 0.20x$$

   Next, we add $0.20x$ and subtract 11 from both sides of the equation to give

   $$x^2 + 0.20x - 11 = 0$$

8. Determine the roots for the equation $2x^2 - x - 15 = 0$ by factorization.

   First, we need to rewrite this equation as the product of two terms. The first term of the polynomial equation, $2x^2$, tells us that factorization is of the form $(2x \pm a)$ and $(x \pm b)$. The last term of the polynomial eauation, -15, means that a and b must be $+5$ and -3, or -5 and $+3$; the first of these combinations is the one that leads to the polynomial equation's second term of $-x$, which makes the results $2x^2 - x - 15 = 0 = (2x + 5)(x - 3)$. To find the roots, we set each of the two terms equal to zero and solve; thus, $2x + 5 = 0$ gives $x = -2.5$ and $x - 3 = 0$ gives $x = +3$.

9. Using the quadratic formula, what are the roots for the equation $3x^2 + 33x - 65 = 0$ to three decimal places?

   The roots of a polynomial equation of the form $ax^2 + bx + c = 0$ are given by

   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

   Substituting in 3 for a, 33 for b, and -6.5 for c and solving gives

   $$x = \frac{-33 \pm \sqrt{33^2 - (4)(3)(-6.5)}}{(2)(3)} = \frac{-33 \pm \sqrt{1089 + 78}}{6} = \frac{-33 \pm \sqrt{1167}}{6} = \frac{-33 \pm 34.16}{6}$$

   The first root is $(-33 + 34.16)/6$ or 0.193 and the second root is $(-33 - 34.16)/6$ or -11.193.

Comfort with scientific notation is important, both in recognizing relative magnitudes and when you enter values in your calculator; these three problems provide practice with scientific notation.

10. Rank the following numbers from smallest-to-largest in magnitude: 9.0×10^{-6}, 8.1×10^{-6}, 1.6×10^{5}, 4.1×10^{-2}, 5.8×10^{4}.

    The order is $8.1 \times 10^{-6} < 9.0 \times 10^{-6} < 4.1 \times 10^{-2} < 5.8 \times 10^{4} < 1.6 \times 10^{5}$

11. Convert the following from decimal notation to scientific notation, or from scientific notation to decimal notation: 0.000139, 452.78, 7.35×10^{-2}, 1.35×10^{5}.

    $0.000139 = 1.39 \times 10^{-4}$, $452.78 = 4.5278 \times 10^{2}$, $7.35 \times 10^{-2} = 0.0735$, $1.35 \times 10^{5} = 135{,}000$

12. What is the value of x if

$$x = \frac{10^{-15}}{3.9 \times 10^{-7}}$$

    The value of x is 2.56×10^{-9}. If your answer is to the wrong power of 10, then you need to review the proper method for entering scientific notation in your calculator.