More Buffer Problems

For some additional practice.

1. A biochemist must prepare a solution to use as a environment for an experiment that involves acid-producing bacteria. The pH of the medium must not change by more than 0.05 pH units for every 0.0010 mol H_3O^+ generated by the organism per liter of medium. A medium consisting of 0.10 M HA and 0.10 M A^- is prepared with a total volume of 1.0 L. Is the buffer capacity of this medium sufficient for the experiment?

   With a volume of 1.0 L and concentrations of 0.10 M HA and 0.10 M A^-, we know that the initial amounts of HA and A^- are 0.10 mol each and the initial pH is

   $$\text{pH}_i = \text{p}K_a + \log\frac{0.10}{0.10} = \text{p}K_a$$

   Adding x moles of a strong acid to a buffer converts an equivalent amount of A^- to HA; thus

   $$\text{pH}_f = \text{p}K_a + \log\frac{0.10 - x}{0.10 + x}$$

   The change in pH upon adding the acid, which is –0.05 (adding an acid makes the pH more acidic so the pH decreases), is

   $$\Delta\text{pH} = \text{pH}_f - \text{pH}_i = -0.05 = \log\frac{0.10 - x}{0.10 + x} - \log\frac{0.10}{0.10} = \log\frac{0.10 - x}{0.10 + x}$$

   Solving for x gives

   $$\frac{0.10 - x}{0.10 + x} = 0.891$$

   $$0.10 - x = 0.0891 + 0.891 \times x$$

   $$x = 5.76 \times 10^{-3}$$

   Thus, we can add as much as 5.76×10^{-3} moles of strong acid and maintain a Δ pH of 0.05; as this is better than the stated requirement, which requires the ability to consumer at least 0.0010 mol of strong acid, the buffer is acceptable.

2. A 1.00-L buffer is prepared that is 0.2000 M in the weak acid, HA, and 0.1500 M in the weak base NaA. The buffer has a pH of 3.35. What is the pK_a for the weak acid?

   To find the weak acid's pK_a we use the buffer equation; thus

   $$3.35 = \text{p}K_a + \log\frac{0.1500}{0.2000}$$

   and solve to obtain a value of 3.447 for the pK_a.

   Is this buffer better at neutralizing strong acid or strong base?

   The buffer has more of its conjugate weak acid, HA, than its conjugate weak base, A^-; thus, the buffer is better at neutralizing a strong base.

   What is the buffer's capacity to neutralize strong acid?

We can add a strong acid until the ratio $\frac{A^-}{HA} = 0.1000$. Adding x moles of strong acid converts an equivalent amount of A^- to HA; thus

$$\frac{\text{mol A}^- - x}{\text{mol HA} + x} = \frac{0.1500 - x}{0.2000 + x} = 0.1000$$

$$0.1500 - x = 0.0200 + 0.1000 \times x$$

$$x = 0.1182$$

Thus, the buffer can neutralize 0.1182 moles of a strong acid.

What is the buffer's capacity to neutralize strong base?

We can add a strong base until the ratio $\frac{A^-}{HA} = 10.00$. Adding x moles of strong base converts an equivalent amount of HA to A^-; thus

$$\frac{\text{mol A}^- + x}{\text{mol HA} - x} = \frac{0.1500 + x}{0.2000 - x} = 10.00$$

$$0.1500 + x = 2.000 - 10.00 \times x$$

$$x = 0.1682$$

Thus, the buffer can neutralize 0.1682 moles of a strong base. Note that this answer is consistent with the expectation that the buffer can neurtralize more strong base than strong acid.

What is the buffer's pH if 00015 mol NaOH is added to 0.5000 L of the buffer?

Because we are working with just half of the buffer, the amounts of weak acid and weak base are 0.1000 mol HA and 0.0750 mol A^-; thus

$$\text{pH} = 3.47 + \log\frac{0.0750 + 0.0015}{0.1000 - 0.0015} = 3.36$$

3. An environmental chemist needs a carbonate buffer of pH 10.00 to study the effects of the acdification of limestone-rich soils. How many grams of Na_2CO_3 must she add to 1.5 L of freshly prepared 0.20 M $NaHCO_3$ to prepare this buffer?

   This buffer is based on HCO_3^- as a weak acid and CO_3^{2-} as a weak base, for which the pK_a is 10.33. The total moles of HCO_3^- initially is

   $$0.20 \text{ M NaHCO}_3 \times 1.5 \text{ L} = 0.30 \text{ mol HCO}_3^-$$

   Substituting into the buffer equations gives

   $$10.00 = 10.33 + \log\frac{\text{mol CO}_3^{2-}}{0.30 \text{ mol HCO}_3^-}$$

   which we solve to find 0.140 mol CO_3^{2-}, or 14.9 of Na_2CO_3.