

Derivation of Least Squares for $y = \beta_0 + \beta_1 x$

$$R = \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

$$R = \sum_{i=1}^n (y_i - b_0 - b_1 x_i)^2$$

$$R = \sum_{i=1}^n (y_i^2 - 2b_0 y_i - 2b_1 x_i y_i + b_0^2 + 2b_0 b_1 x_i + b_1^2 x_i^2)$$

$$R = \sum_{i=1}^n y_i^2 - 2b_0 \sum_{i=1}^n y_i - 2b_1 \sum_{i=1}^n x_i y_i + n b_0^2 + 2b_0 b_1 \sum_{i=1}^n x_i + b_1^2 \sum_{i=1}^n x_i^2$$

$$\frac{\partial R}{\partial b_0} = -2 \sum_{i=1}^n y_i + 2n b_0 + 2b_1 \sum_{i=1}^n x_i = 0$$

$$2n b_0 = 2 \sum_{i=1}^n y_i - 2b_1 \sum_{i=1}^n x_i$$

$$b_0 = \frac{\sum_{i=1}^n y_i - b_1 \sum_{i=1}^n x_i}{n}$$

$$\frac{\partial R}{\partial b_1} = -2 \sum_{i=1}^n x_i y_i + 2b_0 \sum_{i=1}^n x_i + 2b_1 \sum_{i=1}^n x_i^2 = 0$$

$$\sum_{i=1}^n x_i y_i - b_0 \sum_{i=1}^n x_i - b_1 \sum_{i=1}^n x_i^2$$

$$\sum_{i=1}^n x_i y_i - \left(\frac{\sum_{i=1}^n y_i - b_1 \sum_{i=1}^n x_i}{n} \right) \sum_{i=1}^n x_i - b_1 \sum_{i=1}^n x_i^2 = 0$$

$$n \sum_{i=1}^n x_i y_i - \left(\sum_{i=1}^n y_i - b_1 \sum_{i=1}^n x_i \right) \sum_{i=1}^n x_i - n b_1 \sum_{i=1}^n x_i^2 = 0$$

$$n \sum_{i=1}^n x_i y_i - \sum_{i=1}^n x_i \sum_{i=1}^n y_i + b_1 \left(\sum_{i=1}^n x_i \right)^2 - n b_1 \sum_{i=1}^n x_i^2 = 0$$

$$n b_1 \sum_{i=1}^n x_i^2 - b_1 \left(\sum_{i=1}^n x_i \right)^2 = n \sum_{i=1}^n x_i y_i - \sum_{i=1}^n x_i \sum_{i=1}^n y_i$$

$$b_1 \left(n \sum_{i=1}^n x_i^2 - \left(\sum_{i=1}^n x_i \right)^2 \right) = n \sum_{i=1}^n x_i y_i - \sum_{i=1}^n x_i \sum_{i=1}^n y_i$$

$$b_1 = \frac{n \sum_{i=1}^n x_i y_i - \sum_{i=1}^n x_i \sum_{i=1}^n y_i}{n \sum_{i=1}^n x_i^2 - \left(\sum_{i=1}^n x_i \right)^2}$$