Summary of Significance Test

The following symbols are used in this handout:

- $\bar{X}$ is the mean for a set of samples drawn from a population
- s is the standard deviation for a set of samples drawn from population
- μ is the mean for the population
- σ is the standard deviation for the population
- n is the number of samples used to determine $\bar{X}$ and s
- α is the probability of a type I error and $100 \times (1 - \alpha)$ is the confidence level
- ν is the degrees of freedom
- *exp* and *crit* indicates a test statistic's experimental and critical values
- *num* indicates numerator and *denom* indicates denominator
- X_{small} and X_{large} are the smallest and the largest value in a data set, and $X_{near} is the value closest in magnitude to X_{small} or X_{large}

The null hypothesis, H_0 is rejected and the alternative hypothesis, H_A, accepted if the experimental test statistic is greater than or equal to the test statistic's critical value. Examples are from Chapter 4 of *Analytical Chemistry 2.1*.

Comparing $\bar{X}$ to μ Using a t-Test

- $H_0 : \bar{X} = \mu$
- $t_{exp} = \frac{|\mu - \bar{X}|\sqrt{n}}{s}$
- $t_{crit} = t(\alpha, \nu)$
- $\nu = n - 1$
- see Example 4.16

Comparing s^2 to σ^2 Using an F-Test

- $H_0 : s^2 = \sigma^2$
- $F_{exp} = \frac{s^2}{\sigma^2}(\text{ if } s > \sigma)$ or $F_{exp} = \frac{\sigma^2}{s^2}(\text{ if } \sigma > s)$
- $F_{crit} = F(\alpha, \nu_{num}, \nu_{denom})$
- $\nu = n - 1$ for s and $\nu = \infty$ for σ
- see Example 4.17

Comparing s_a^2 to s_b^2 Using an F-Test

- $H_0 : s_a^2 = s_b^2$
- $F_{exp} = \frac{s_a^2}{s_b^2}(\text{ if } s_a > s_b)$ or $F_{exp} = \frac{s_b^2}{s_a^2}(\text{ if } s_b > s_a)$
- $F_{crit} = F(\alpha, \nu_{num}, \nu_{denom})$
- $\nu = n - 1$ for both s_a and s_b
- see Example 4.18

Comparing $\bar{X}_a$ to $\bar{X}_b$ Using an Unpaired t-Test

When each of the two sets of data are independent of each other—that is, when the samples used to calculate $\bar{X}_a$ and not the same samples used to calculate $\bar{X}_b$—then the data are unpaired. There are two approaches for working with an unpaired t-Test depending on whether or not you can pool the standard deviations for the two sets of sample, which we determine using an F-Test for s_a^2 and s_b^2, as described above. If the two standard deviations are significantly different from each other (or if there are experimental reasons why the standard deviations do not come from the same population), then the t-Test is described this way

- $H_0 : \bar{X}_a = \bar{X}_b$
- $t_{exp} = \frac{|\bar{X}_a - \bar{X}_b|}{\sqrt{\frac{s_a^2}{n_a} + \frac{s_b^2}{n_b}}}$
- $t_{crit} = t(\alpha, \nu)$
- $\nu = \frac{\left[\frac{s_a^2}{n_a} + \frac{s_b^2}{n_b}\right]^2}{\frac{\left(\frac{s_a^2}{n_a}\right)^2}{n_a+1} + \frac{\left(\frac{s_b^2}{n_b}\right)^2}{n_b+1}} - 2$
- see Example 4.20

If we do not find a significant difference between the two standard deviations, the t-Test is described this way

- $H_0 : \bar{X}_a = \bar{X}_b$
- $t_{exp} = \frac{|\bar{X}_a - \bar{X}_b|}{s_{pooled}} \times \sqrt{\frac{n_a \times n_b}{n_a + n_b}}$
- $s_{pooled} = \sqrt{\frac{(n_a-1)s_a^2 + (n_b-1)s_b^2}{n_a + n_b - 2}}$
- $t_{crit} = t(\alpha, \nu)$
- $\nu = n_a + n_b - 2$
- see Example 4.19

Comparing $\bar{X}_a$ to $\bar{X}_b$ Using a Paired t-Test

When the two data sets are not independent—that is, the same samples or same sampling locations are used for both data sets—then the data are considered paired. This is common, for example, with clinical studies if the same patients are tested before and after treatment, in environmental studies if the same sites are tested before and after remediation, or when comparing two analytical methods using a common set of samples. In this case the data are reduced to a single set of differences

$$d_i = (X_a)_i - (X_a)_i$$

and the t-Test is described as

- $H_0 : \bar{d} = 0$
- $t_{exp} = \frac{|\bar{d}|\sqrt{n}}{s_d}$
- $t_{crit} = t(\alpha, \nu)$
- $\nu = n - 1$
- see Example 4.21

Using Dixon's Q-Test to Detect Outliers

Use caution when testing for outliers. There are other tests for outliers not covered here.

- $H_0 : X_{small}$ is not an outlier or $H_0 : X_{large}$ is not an outlier
- $Q_{exp} = \frac{X_{near} - X_{small}}{X_{large} - X_{small}}$ or $Q_{exp} = \frac{X_{large} - X_{near}}{X_{large} - X_{small}}$
- $Q_{crit} = Q(\alpha, \nu)$
- $\nu = n$
- see Example 4.22