

Short Problem Set 5

Fit the model $y = \beta_0 + \beta_1 x$ to the following data

$x : 0, 1.00, 2.00, 3.00, 4.00, 5.00$

$y : 0, 0.94, 2.15, 3.19, 3.70, 4.21$

and report the coefficient of determination, the standard deviation about the regression, and an analysis of variance. You may use Excel to help you with the various summations needed to complete the calculations. Prepare a plot and add the regression line. Briefly discuss whether the regression model is a suitable model for this data.

Answer

The following table contains all the information we will need:

	x	y	xy	x^2	\hat{y}	$(\hat{y} - \bar{y})^2$	$(y - \bar{y})^2$	$(y - \hat{y})^2$
	0	0	0	0	0.196	4.705	5.593	0.038
	1.00	0.94	0.94	1.00	1.063	1.694	2.031	0.015
	2.00	2.15	4.30	4.00	1.931	0.188	0.046	0.048
	3.00	3.19	9.57	9.00	2.799	0.188	0.681	0.153
	4.00	3.70	14.80	16.00	3.667	1.694	1.782	0.001
	5.00	4.21	21.05	25.00	4.534	4.705	3.464	0.105
sum	15.00	14.19	50.66	55.00	—	13.176	13.537	0.361
avg	—	2.365	—	—	—	—	—	—

$$b_1 = \frac{6 \times 50.66 - 15.00 \times 14.19}{6 \times 55.00 - 15.00^2} = 0.868$$

$$b_0 = \frac{14.19 - 0.868 \times 15.00}{6} = 0.196$$

$$y = 0.196 + 0.868x$$

$$r^2 = \frac{13.176}{13.537} = 0.973$$

$$s_r = \sqrt{\frac{0.361}{6 - 2}} = 0.300$$

$$MS_{regression} = \frac{13.176}{1} = 13.176$$

$$MS_{residual} = \frac{0.361}{6 - 2} = 0.0902$$

$$F_{exp} = \frac{13.176}{0.0902} = 146.1$$

$$F_{crit} = F(0.05, 1, 4) = 7.709$$

All three methods for evaluating the regression suggest that there is a correlation between y and x ; however, a glance at the data and the model suggests that there might be curvature in the data not explained by the model.

